2D Electromagnetic Scattering Solution Using EFG Meshless Method and Differential Evolution Optimization Algorithm

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This paper presents the use of the Differential Evolution algorithm as an optimization method for 2D electromagnetic scattering problem solved by interpolating Element-Free Galerkin (EFG) meshless method. It is considered the TMz plane wave scattering by a z-infinite dielectric cylinder. The numeric and analytic solutions are compared by using the L^2 norm error. The Differential Evolution method is applied in order to find good sets of interpolating EFG parameters that minimize the L^2 norm error.

***Index Terms*—EFG, electromagnetic scattering, computational electromagnetics, meshless methods, optimization methods.**

I. INTRODUCTION

MESHLESS methods have been used to numerically solve partial differential equations associated to practical problems in several science areas like structure and fluid mechanics and, more recently, electromagnetic problems [1]–[4]. This class of methods does not use a previously created mesh to discretize the domain so that it is specially suitable to solve problems involving complex or variable geometries.

The Element-Free Galerkin Method (EFG) [5] is currently one of the most popular meshless methods. Additionally to the mesh independence benefits, it features a symmetric resultant linear system matrix, which is important since this matrix has to be inverted and hence cannot be singular. EFG also presents ease of discretization and independence of integration in the weak form. Recent works use EFG and the Interpolating Moving Least Square method (IMLS) to obtain shape functions that satisfy the Kronecker Delta property [2]–[4].

Despite the good characteristics of the EFG-IMLS method, it involves choosing appropriated parameters values (e.g. ABC radius, number of nodes/integration cells) that lead to accurate results. So, it is opportune to use optimization methods to find a set of optimal values for these parameters. In this scenario, the Differential Evolution (DE) method appears as a good option due to its main features: ease of implementation, fast convergence (when compared to other evolutionary methods) and ability to work with non differentiable non convex objective functions [6].

This work applies the DE method to optimize the electromagnetic scattering problem involving a TMz plane wave and a z-infinite dielectric cylinder solved by EFG-IMLS method.

II. 2D EM SCATTERING AND EFG-IMLS FORMULATIONS

The scattering problem under investigation consists in an infinite cylinder Ω_1 along the z-axis direction, formed by a dielectric material of relative permittivity ϵ_r and which radius is ρ_r, as shown in Fig. 1. This scatterer is located in free space (Ω_2) and is illuminated by a TMz plane wave. The solution domain, $\Omega = \Omega_1 \cup \Omega_2$, is limited by the boundary Γ. The total electric field E, which has only the z-component, is calculated by the bi-dimensional scalar Helmholtz equation $\nabla^2 E_z + k_0^2 E_z = 0$, where k_0 is the vacuum wave number [3].

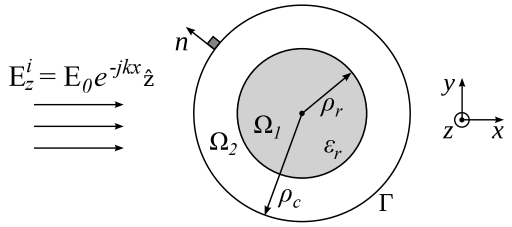

Fig. 1. 2D scattering problem.

The weak form for the problem can be obtained by multiplying a test function w by the residue of Helmholtz equation. The product is integrated on Ω and the Gauss' divergence theorem is applied to the result. It is necessary to establish a global boundary Γ, located at a distance away from the scatterer, where an Absorbing Boundary Condition (ABC) is imposed. Using a first order Bayliss-Turkel ABC, the weak form is [3]:

$$\int_\Omega \left[\nabla w \left(\frac{1}{\mu_r} \nabla E_z \right) - k_0^2 \epsilon_r w E_z \right] d\Omega + \int_\Gamma \gamma w E_z d\Gamma = \int_\Gamma q w d\Gamma, \quad (1)$$

where $\gamma = 1/\mu_r \, [jk_0 + 1/(2\rho_r)]$, $q = 1/\mu_r \left(\nabla E_z^i \cdot \mathbf{n} + \gamma E_z^i \right)$, μ_r is the relative permeability, E_z^i is the incident electric field and $\mathbf{n}$ is the unitary vector which direction is perpendicular to Γ,

In the EFG approach, NN nodes located by $\mathbf{x}_i = (x, y) \in \Omega$ are distributed over the domain. To each node must be associated a shape function Φ_i so that $\Phi_i = 0$ for the whole domain Ω, except a region near $\mathbf{x}_i$. Thus, the unknown function E_z can be approximated by the trial function $E^h(\mathbf{x})$ [7]:

$$E^h(\mathbf{x}) = \sum_{i=1}^{NN} \Phi_i(\mathbf{x}) v_i, \quad (2)$$

where v_i are unknown coefficients. In EFG-IMLS, Φ_i is determined by using the local approximation:

$$E^h(\mathbf{x}, \mathbf{x}_i) = \sum_{i=1}^{m} p_i(\mathbf{x}_i) a_i(\mathbf{x}) \equiv \mathbf{p}^T(\mathbf{x}_i) \mathbf{a}(\mathbf{x}), \quad (3)$$

where $\mathbf{p}^T(\mathbf{x})$=$[1, x, y]$ is a polynomial base with $m = 3$ monomial terms, and $\mathbf{a}(\mathbf{x})$ are unknown polynomial coefficients. In the IMLS approximation used in this work, $\mathbf{a}(\mathbf{x})$ are determined by minimizing a weighted discrete L^2 norm, for which weight function is $W(r) = 1/(r^n + \beta^n)$, where β is a constant which value must be small enough to ensure no division by zero, n

is a constant adjusted to improve the accuracy, and r is the support radius of influence domain for each node [2], [3].

III. Differential Evolution optimization

The DE algorithm is a stochastic population-based function minimizer suitable for non-differentiable functions and capable of fast convergence. Its basic operators are defined as follows.

Let $X_G = \{\mathbf{x}_{G,i}, i = 1, \ldots, NP\}$ an initial population formed by NP individuals $\mathbf{x}_{G,i}$, that is, solution candidates vectors, each with n parameters. The mutation operator adds the difference between any two randomly selected individuals (the difference vector) to a third individual, also randomly picked, in order to create a mutated population, V_{G+1}, that is, $\mathbf{v}_{G+1,i} = \mathbf{x}_{G,r_1} + F(\mathbf{x}_{G,r_2} - \mathbf{x}_{G,r_3}), r_1 \neq r_2 \neq r_3 \neq i \in [1, NP]$ where $\mathbf{v}_{G+1,i}$ is a mutant individual, and F is a scale factor applied to the difference vector length [6].

The population diversity can be increased by combining individuals from X_G with others from V_{G+1}. This is the cross-over operation which has a CR probability of occurring [6].

$$\mathbf{u}_{G+1,i,j} = \begin{cases} \mathbf{v}_{G+1,i,j} & \text{if } \mathcal{U}_{[0,1]} \leq CR \ \vee \ j = \delta_i \\ \mathbf{x}_{G,i,j} & \text{otherwise} \end{cases} \tag{4}$$

where $\delta_i \in [1, n]$ is a randomly selected index and $\mathcal{U}_{[0,1]}$ is a uniformly distributed random number between 0 and 1. Here, the operators are defined as in the DE basic form. Other DE schemes may use different operator definitions [6].

The objective function $f(\cdot)$ is evaluated for each $\mathbf{u}_{G+1,i}$ and the result is compared to that obtained by evaluating $f(\mathbf{x}_{G,i})$. If $f(\mathbf{x}_{G,i}) > f(\mathbf{u}_{G+1,i})$, $\mathbf{u}_{G+1,i}$ replaces the corresponding individual $\mathbf{x}_{G,i}$ in the initial population for the next generation [6]. The problem restrictions where treated by penalty method.

After executing a given number of iterations, the algorithm returns the minimum value found for $f(\cdot)$ and the corresponding set of optimal parameters.

IV. Results

Aiming to verify the pertinence of the optimization strategy presented in this work, the cylinder described in Section II is analysed in three different radius: 0.1λ, 0.3λ and 0.5λ, with the vacuum wavelength λ=1m. The cylinder material relative permittivity is ϵ_r=2. The numeric solution calculated by EFG-IMLS algorithm is checked against the analytical solution (AS) by the following L^2 norm error over the whole domain Ω [3]:

$$EL^2 = \sqrt{\iint_\Omega \left| E_z^{\text{EFG-IMLS}} - E_z^{\text{AS}} \right|^2 d\Omega} \Bigg/ \sqrt{\iint_\Omega \left| E_z^{\text{AS}} \right|^2 d\Omega} \times 100\% \tag{5}$$

Three EFG-IMLS parameters were defined to be optimized: the ABC radius ρ_c, the number of nodes employed and the integration cells number. In this work all simulation are carried out using four Gauss integration points in each cell. The three optimized parameters are related to each other and are decisive to the result accuracy and performance requirements.

For optimization process, different values of these parameters are placed in vector-individuals that form the populations in each DE generation. When the minimum error is found, the corresponding vector-individual is the optimized set of parameters.

TABLE I
EL^2 NORM PERCENT VALUES CALCULATED BY DE-EFG ALGORITHM

Cyl. radius	Optimal	Maximun	Mean	Std. deviation
0.1λ	0.2590	0.3342	0.2927	0.0123
0.3λ	1.1286	1.8918	1.5459	0.1462
0.5λ	4.0197	5.2672	4.7113	0.2837

TABLE II
OPTIMAL EFG-IMLS PARAMETERS FOUND BY DE ALGORITHM

Cyl. radius	ABC radius	Number of nodes	Number of cells
0.1λ	2.4853	340	10572
0.3λ	1.9983	340	5544
0.5λ	1.8394	411	7876

The DE algorithm is set to run 50 iterations using populations of 15 individuals, then it is executed 100 times for each case. F varies randomly in $[0.5, 1]$ and CR is set to 0.5.

Table I shows the summarized results. For each case, columns 2, 3 and 4 have the optimal, maximum and mean EL^2 values. Column 5 has the standard deviation for the samples. Table II contains the optimal parameters for each analysed case.

This data shows that coupling DE optimizer with the EFG-based algorithm makes possible to find combinations of suitable parameters to solve the electromagnetic scattering problem using the EFG-IMLS method for different cylinder radius, while keeping acceptable error values, when comparing to the analytical solution. For Case 2, e.g., in 95% of DE-EFG executions, EL^2 norm is expected to be less than 1.69%.

V. Conclusion

This work presents a technique to choose optimal combinations of parameters values to EFG-IMLS meshless method applied in the solution of 2D electromagnetic scattering in order to obtain accurate numerical results. The DE algorithm proved to be suitable to this task, since it was possible to obtain different sets of parameters values which lead to EL^2 norm values less than 5.0%, in the worst case analysed.

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